Comparison of FK and SPAC Methods in Determining Dispersion Curves From Passive Surface Waves

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Outline

- Passive Surface Waves
- Dispersion Analysis Methods
- Comparison of Simulated Data
- Field Data Examples
- Conclusion
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Passive Surface Waves – Procedure
Passive Surface Waves – Array

- **2D Array** (Triangle; Circle; Cross; L shape)
  - Allow waves coming from any directions

- **1D Array** (Linear)
  - Assume waves coming
    - (1) Along the direction of array (roadside) or
    - (2) From all directions (omni-directional)

\[ V_m = V_R \]  
\[ V_m = \infty \]  
\[ V_m = V_R \]  
\[ V_R < V_m < \infty \]

- \( V_m \): measured velocity; \( V_R \): the true phase velocity
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Dispersion Analysis Methods

There are two commonly used methods to measure the phase velocity of passive surface waves:

1. **SPAC / ESPAC**
   Fitting the spatial autocorrelation coefficient

2. **FK**
   Azimuth scanning in FK, FV, or FP domain
SPAC – Coherency

Provided a set of geophones azimuthally deployed around the circle, for a pair of geophones, the coherency:

\[ C_{j,c}(f) = \exp[i r_{j,c} k \cos(\theta_{j,c} - \phi)] \]

Where \( r_{j,c} \) is the displacement of the \( j \)th geophone relative to the center at azimuthal angle \( \theta_{j,c} \), \( k \) is the spatial wavenumber at frequency \( f \), and \( \phi \) is the azimuth of propagation of a single plane wave across the array.
SPAC – Azimuthal Average Coherency

If the number of pairs of geophones approach to infinite, the spatial autocorrelation coefficient (azimuthal average of the coherency) can be expressed as:

\[ c(f) = \frac{1}{2\pi} \int_0^{2\pi} \exp[irk \cos(\theta - \phi)] d\theta = J_0(rk) = J_0 \left( \frac{2\pi fr}{v(f)} \right) \]

where \( J_0 \) is the Bessel function of the first kind of zero order.

(1) The spatial autocorrelation coefficient is a function of the phase velocity and frequency; and

(2) The phase velocity \( v(f) \) can be derived by fitting the spatial autocorrelation coefficient \( c(f) \), which is calculated from the recorded signals, to the Bessel functions.

Note that this equation also demonstrates that the linear array is allowed if the passive waves come from all directions, i.e. integrated around the circle over \( \phi \) with \( \theta \) fixed.
ESPAC – Extended SPAC

Sometimes, it is difficult to deploy a regular array, such as circle, triangle.

SPAC has been improved to ESPAC (Extended Spatial Autocorrelation), which allows the irregular array.
FK Method

FK method computes a 2D frequency-wavenumber power spectrum from an array, it has several forms, here is the maximum likelihood,

\[
P(f) = \left\{ \sum_{i=1}^{N} \sum_{j=1}^{N} \phi_{ij}(f) \exp[i k_x (x_i - x_j) + i k_y (y_i - y_j)] \right\}^{-1}
\]

Where \( f \): frequency; \((x_i, y_i)\) coordinates of receivers i and j; \((k_x, k_y)\) the wave number vector; \( \phi_{ij}(f) \) the correlation of signals at receiver i and j.

For a given frequency \( f_0 \), the location \( k_0 = (k_{x0}, k_{y0}) \) of the maximum of semblance provides an estimate of velocity and azimuth of the wave traveling across the array, i.e.

\[
V_0 = \frac{2 \pi f_0}{k_0} = \frac{2 \pi f_0}{\sqrt{(k_{x0}^2 + k_{y0}^2)}} , \quad \theta = \tan^{-1} \left( \frac{k_{x0}}{k_{y0}} \right)
\]
FK – Semblance Images

For a given frequency, scan all directions with 2d wavenumber

(After Maxime Claprood)
FK – Azimuth Scan

FK transform is conducted for all directions through the geophone array (azimuth scan), the final dispersion image is the summation of all of them.
FK – Dispersion Picks for Linear Array

For the linear array, the dispersion curve of the passive surface wave should be picked along the envelope of the dispersion image with the slowest velocity.
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Simulation – Model

Model

Dispersion curve

Dispersion Curve (Circle)
Simulation – Sources

200 sources randomly distributed with different strength
Simulation – Arrays
Simulation – Records
Dispersion Image Comparison – Circle

SAGEEP 2010
Dispersion Image Comparison – Triangle
Dispersion Image Comparison – L-Shape
Dispersion Image Comparison – Linear
Dispersion Curve Comparison

Dispersion Curves (Linear)

Dispersion Curve (L-Shape)

Dispersion Curve (Triangle)

Dispersion Curve (Circle)
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Field Example – 1

- Circular array
- 24m diameter
- 10 geophones
- 7 min. length
- 1 record
Field Example – 2

- L shape
- 100ft per side
- 11 geophones
- 30 sec. length
- 10 records
Field Example – 3

- Linear array
- 55m length
- 12 geophones
- 30 sec. length
- 10 records
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Conclusion

- SPAC shows higher resolution for larger frequency range
- FK can provide reliable results for higher frequency but has lower resolution for the lower frequency
- SPAC is recommended when waves come from all directions no matter which array is used
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