S-Wave Velocity Profiling by Joint Inversion of Microtremor Dispersion Curve and Horizontal-to-Vertical (H/V) Spectrum

by Hiroshi Arai and Kohji Tokimatsu

Abstract A joint inverse analysis using both microtremor dispersion curve and horizontal-to-vertical (H/V) spectrum is proposed for estimating the S-wave velocity ($V_S$) profiles of subsurface soils. In the inversion, both microtremor dispersion and H/V data are assumed to be the Rayleigh-wave dispersion curve and the surface (both Rayleigh and Love) wave H/V spectrum that have been theoretically derived by taking into account the effects of their fundamental and higher modes. The proposed joint inversion as well as the conventional one using dispersion data alone is performed at four sites where shallow $V_S$ profiles down to engineering bedrock are available. The $V_S$ profiles estimated by the proposed joint inversion are more consistent with available down-hole velocity logs than those by the conventional method. In particular, the proposed inversion shows significant improvement in estimating bedrock $V_S$ structures compared to the conventional inversion. Sensitivity analyses indicate that the surface-wave H/V ratio is sensitive to the bedrock $V_S$ structure more than the Rayleigh-wave phase velocity, confirming that the proposed joint inversion including H/V spectrum is promising.

Introduction

The S-wave velocity ($V_S$) structure of sedimentary deposits is one of the key components controlling site-dependent strong ground motions and resulting geotechnical problems and structural damage during earthquakes. Knowing the $V_S$ profile at a site is therefore fundamental to the prevention or mitigation of earthquake disasters and can be determined by the conventional geophysical or geotechnical methods using boreholes. However, when exploring two- and three-dimensional or deep underground $V_S$ profiles, it often becomes difficult to use the conventional methods because of their cost and the time required. As an economical and practical substitute, microtremor measurements, which can be readily performed on the ground surface without drilling any borehole, have therefore often been used.

A number of previous studies have shown that the frequency–wave number ($f$-$k$) spectral analysis (Capon, 1969) and the spatial autocorrelation (SAC) analysis (Aki, 1957) for microtremor vertical components measured with arrays of sensors can yield the dispersion characteristics of Rayleigh waves. In addition, it has been revealed that the inverse analysis of microtremor dispersion data successfully results in the $V_S$ profile of a sedimentary deposit (e.g., Horike, 1985; Matsushima and Okada, 1990a; Tokimatsu et al., 1992). These studies indicate that the array observation of microtremor vertical motions is an effective tool for evaluating $V_S$ structure at a sedimentary site (e.g., Horike, 1993; Tokimatsu, 1997).

Figure 1 illustrates a typical example of sensor array configurations used for microtremor measurements at a site. Several vertical component sensors are placed at equal spacing along the circumference of an imaginary circle drawn on the ground surface, with one sensor at the center of the circle. For such a circular array or any other array configuration, the effective wavelength range yielding reliable phase velocities from the $f$-$k$ analysis is given approximately as (e.g., Asten and Henstridge, 1984; Tokimatsu, 1997)

$$L_{\text{max}} > \lambda_{\text{max}}/3$$

and

$$L_{\text{min}} < \lambda_{\text{min}}/2$$

where $L_{\text{max}}$ and $L_{\text{min}}$ are the maximum and minimum inter-sensor distances, and $\lambda_{\text{max}}$ and $\lambda_{\text{min}}$ are the maximum and minimum effective wavelengths, respectively. This means that the effective wavelength for any array is limited to the value in between twice the minimum sensor distance and three times the maximum sensor distance of the array. This corresponds to 1–3 times the diameter for a circular array. Therefore, microtremor measurements must be repeated with different array sizes until the phase velocities within the wavelength range of interest are derived. The phase velocity thus obtained is strongly affected by the S-wave ve-
velocity at a depth of one-third of its wavelength (e.g., Asten and Henstridge, 1984). Thus, the maximum depth that can be estimated through the inversion of microtremor dispersion data derived from the f-k method is roughly equal to the maximum sensor distance of the array or to the diameter of the circular array.

It is suggested that the effective wavelength and the maximum explorative depth for the SAC method could extend about 1.5 to 2 times those for the f-k method (e.g., Okada, 2004). In the SAC method, a circular array configuration with equal intersensor spacing (regular polygon array) is mainly used because it is easy to compute the azimuthal average of the spatial autocorrelation function in various wave propagation conditions (e.g., Okada, 2004). In reality, however, such a regular polygon array is often impractical in urban areas because of its restricted sensor configuration.

To estimate the deep $V_S$ structure at a site from inversion of dispersion data, phase velocities of low frequencies with large wavelengths are required. This calls for a large aperture array; however, it may be difficult and troublesome to observe microtremors with large aperture arrays, compared with small aperture arrays.

One of the possible alternatives to bypass this difficulty is to combine other geophysical information reflecting deep $V_S$ structure with microtremor dispersion data derived from small aperture arrays. The studies of Nakamura (1989) have indicated that the horizontal-to-vertical (H/V) spectral ratio of microtremors, conveniently observed at a site with only one three-component sensor, may approximate the amplification factor of the site for vertically incident $S$ wave. After a number of studies on the microtremor H/V spectra, it has been widely confirmed that the peak frequency of the microtremor H/V spectrum at a site corresponds nearly to the natural site frequency for the $S$ wave when the $V_S$ structure of the site has uniquely a high-contrast layer boundary. Based on the array observations, however, Tokimatsu and Miyadera (1992) revealed that the variation of microtremor H/V ratio with frequency corresponds to that of the fundamental-mode of Rayleigh wave for the $V_S$ profile at the site. Subsequent studies have shown that, given $V_S$ values of all layers, the variation of thickness of the layers can be estimated through the inversion with microtremor H/V spectrum assuming that it reflects the fundamental-mode of Rayleigh wave (e.g., Yamanaka et al., 1994; Tokimatsu et al., 1998; Fäh et al., 2001). In addition, based on the theory proposed by Harkrider (1964), Arai and Tokimatsu (2004) presented theoretical formulas for simulating microtremor H/V spectra in which the effects of fundamental and higher modes can be considered for both Rayleigh and Love waves. Using the theoretical formulas, they also presented an inverse analysis of microtremor H/V data for estimating shallow $S$-wave velocity profile down to a depth of engineering bedrock with $V_S$ of about 700 m/sec, provided that either $V_S$ values or thicknesses are known. The inverted $S$-wave velocity profiles were found to be consistent with the available velocity logs at the sites, indicating that the inversion of microtremor H/V spectrum is promising for estimating the $V_S$ profile down to the engineering bedrock, provided that either layer thickness or $S$-wave velocity is known. Fäh et al. (2001) have also made a similar suggestion.

The above discussions indicate that, during microtremor array measurements, the H/V spectrum at low frequencies, where the phase velocity is difficult to determine without a large aperture array, can easily be obtained by replacing the vertical sensor at the center with one three-component sensor. A joint inversion of both microtremor dispersion curve
and $H/V$ spectrum could therefore serve as a convenient and promising tool to estimate the $V_S$ profile down to the engineering bedrock (see Fig. 1). Based on a similar concept, Scherbaum et al. (2003) and Parolai et al. (2005) have also addressed the importance of such a joint inversion to estimate correctly the $V_S$ profile down to the bedrock from the microtremor method. Tokimatsu and Tamura (1992) have proposed a similar joint inversion using both dispersion curve and $H/V$ amplitude ratios of particle orbits for multiple-mode Rayleigh waves generated from a point source acting on the ground surface and indicated that their proposed inversion can enhance the reliability of inverted $V_S$ structure; however, the method cannot be directly applied to an inversion of microtremor data as microtremors contain both Rayleigh and Love waves.

The objectives of this article are to introduce the joint inversion analysis using both the dispersion curve and the $H/V$ spectrum of microtremors considering the effects of fundamental and higher modes of Rayleigh and Love waves to estimate the $V_S$ profile and to examine the reliability and accuracy of the proposed inversion methodology.

### Joint Inversion Methodology of Dispersion Curve and $H/V$ Spectrum

The soil layer model used in this article is assumed to be a semiinfinite elastic medium consisting of $N$ parallel, solid, homogeneous, isotropic layers. Each layer is characterized by its thickness, $H$, density, $\rho$, $P$-wave velocity, $V_P$, and $S$-wave velocity, $V_S$ (see Fig. 2). Sensitivity analyses have shown that $V_S$ and $H$ have stronger influence than $V_P$ and $\rho$ on Rayleigh-wave dispersion curves and surface-wave $H/V$ spectra (e.g., Tsuboi and Saito, 1983; Horike, 1985; Arai and Tokimatsu, 2004). The joint inversion of microtremor dispersion curve and $H/V$ spectrum can therefore be performed only with the $V_S$ and $H$ of the deposit, keeping the values of $V_P$ and $\rho$ constant. Thus, the total number of unknown parameters, $J$, is $2N - 1$.

Although various nonlinear inversion methodologies to estimate $S$-wave velocity profiles using dispersion curves of Rayleigh waves have been presented (e.g., Horike, 1985; Matsushima and Okada, 1990a; Tokimatsu et al., 1992; Yuan and Nazarian, 1993), they may not be directly applicable to the microtremor $H/V$ spectrum. The $H/V$ inversion is highly at risk of divergence during its iteration process because its variation with frequency and residual-norm space are more complicated than those of dispersion curves. The inversion method adopted in this study is, therefore, an extended version of that for the microtremor $H/V$ spectrum introduced by Arai and Tokimatsu (2004). An outline of the inversion method follows.

If the number of observed phase velocities of microtremor vertical motions, $c_m$, and the $H/V$ spectral ratios of microtremors, $(H/V)_m$, are given as $I_{HR}$ and $I_{HV}$, respectively, the goal of the inversion process is to find a soil layer model that satisfies the following generalized least-squares equation:

$$F = F_R + F_{HV} = \frac{w^2_R}{I_{HR}} \sum_{i=1}^{I_{HR}} \left( \frac{c_{mu} - c_{Ri}}{c_{mu}} \right)^2 + \frac{w^2_{HV}}{I_{HV}} \sum_{i=1}^{I_{HV}} \left( \frac{(H/V)_{mu} - (H/V)_{Si}}{(H/V)_{mu}} \right)^2 \to \min,$$

where $c_{Ri}$ and $(H/V)_{Si}$ are the theoretical phase velocity of Rayleigh waves (Tokimatsu et al., 1992) and the $H/V$ spectral ratio of surface waves at a frequency $f_i$ computed for a soil layer model considering the effects of fundamental and
higher modes determined by equation (20) in the article by Arai and Tokimatsu (2004), in which the value of Rayleigh-love wave amplitude ratio for horizontal motions (RL) is assigned to 0.7 at all frequencies, based on the studies by Matsushima and Okada (1990b) and Arai and Tokimatsu (2000). \( w_k \) and \( w_{HV} \) are weighting factors for the dispersion curve and H/V spectrum. The H/V spectral ratio of microtremors at a frequency \( f \), \( (H/V)_{mf}(f) \), in equation (3) is defined as

\[
(H/V)_{mf}(f) = \frac{P_{NS}(f) + P_{EW}(f)}{P_{UD}(f)},
\]

where \( P_{UD}(f) \) is the Fourier power spectrum of the vertical motion, and \( P_{NS}(f) \) and \( P_{EW}(f) \) are those of two orthogonal horizontal motions. The power spectra are determined by using the direct segment method (Capon, 1969) without any smoothing window (e.g., Arai and Tokimatsu, 2004). No other smoothing technique is used in this study.

To solve equation (3), the generalized (nonlinear) least-squares method (e.g., Dorman and Ewing, 1962; Wiggins, 1972) is used because of the highly nonlinear nature of the problem. In such a nonlinear problem, several iterations are necessary before the final soil layer model is identified (see Fig. 2). To initiate the inversion process, an initial soil profile \( P^{(0)} \) is assumed. \( P^{(0)} \) is a column vector consisting of \( 2N - 1 \) (= \( J \)) elements, \( p^{(0)}_j \), each of which consists of either thickness or S-wave velocity of each layer. After the \( k \)th iteration, the soil profile is updated to \( P^{(k)} \). In the nonlinear inversion, the system is approximately linearized at each iteration around the soil profile \( P^{(k)} \). The governing equation of the nonlinear inversion problem is then expressed in a matrix form as

\[
W^{adj} \Delta y^{(k)} = W^{adj} A^{(k)} \Delta x^{(k)}
\]

in which \( \Delta y^{(k)} \) is a column vector whose \( (I_R + I_{HV}) \) elements are the normalized misfits between the observed and theoretical values with weighting factors, \( w_R (c_{nu} - c^{(k)}_{nu})/c_{nu} \) and \( w_{HV} (H/V)^{mf}_{nu} - (H/V^{(k)}_{nu})/(H/V)_{nu} \). \( A^{(k)} \) is an \( (I_R + I_{HV}) \times J \) matrix whose elements are the normalized (non-dimensional) partial derivative of the theoretical phase velocity and H/V ratio for each model parameter, \( w_R \frac{[p^{(k)}_j/c_{nu}]}{[\partial (H/V^{(k)}_{nu})/\partial p]} \) and \( w_{HV} \frac{[p^{(k)}_j/(H/V)^{mf}_{nu}]}{[\partial ((H/V)^{mf}_{nu})/\partial p]} \), respectively, \( \Delta x^{(k)} \) is defined as a normalized modification column vector whose \( (J) \) elements are the modification ratios of the parameter, \( \Delta p^{(k)}_j/p^{(k)}_j \), and \( W^{adj} \) is an \( (I_R + I_{HV}) \times (I_R + I_{HV}) \) diagonal matrix whose diagonal elements are weighting factors determined by the adaptive biweight estimation method (Tukey, 1974) for all the normalized misfits.

In solving equation (5), the singular value decomposition method (Golub and Reinsch, 1970) combined with the modified Marquardt’s technique (Marquardt, 1963; Fletcher, 1971) is used, and the updated soil layer model for the \( (k + 1) \)-th iteration, \( P^{(k+1)} \), is then determined. The iteration procedure is repeated until the error ratio criterion, \( \varepsilon \), which is the root mean of the sum of the squares of the normalized misfit, that is, \( \sqrt{\varepsilon} \), converge into an acceptably small value and the updated soil layer model is considered to the final solution (see Fig. 2). At the final stage of the inversion when the \( \varepsilon \) value gets small without any divergence, the weighting factors by the biweight estimation (the diagonal elements of \( W^{adj} \) matrix) and the Marquardt’s factors in the modified Marquardt’s technique (e.g., Fletcher, 1971) are set equal 1 and 0, respectively, in order to switch the inversion back to the ordinary one.

Joint Inversion of Microtremor Dispersion Curve and H/V Spectrum for \( V_S \) Profiling

Array Measurements of Microtremors

To examine the effectiveness of the proposed joint inversion, microtremor measurements with circular sensor arrays shown in Figure 1 were performed at four sites in the cities of Kushiro, Odawara, and Tokyo, Japan, subsequently called sites A, B, C, and D, where boring and \( PS \) logging data of shallow soil layers are available down to depths of 20 m, 98 m, 77 m, and 100 m, respectively (e.g., Ishihara et al., 1989; Kashima et al., 1994; Matsunaga et al., 1994; Sato et al., 1998). Sites A and C are located in Kushiro city, and sites B and D are in Odawara and Tokyo, respectively. The \( PS \) logging was performed by the down-hole method using the borehole at each site. At sites A, B, and D, the boreholes reach down to engineering bedrock with \( V_S \) of about 600–800 m/sec. The structures of deep soil layers at each site below the engineering bedrock down to seismic ones with \( V_S \) of about 3 km/sec (Tables 1–4) are inferred using avail-
section 200–3000 1.9 1800 700
100–1500 1.9–2.0 1700–2400 600–1000
200–3000 2.1 3600 1900
>3000 2.3 5600 3200

Data from Kushiro Office in Hokkaido Association of Architects and Building Engineers (1989); Miyakoshi and Okada (1996).

Table 4
Deep Ground Structure Inferred from Results of Seismic Explosion Survey near Site D in Tokyo, Japan

<table>
<thead>
<tr>
<th>Depth (m)</th>
<th>ρ (t/m³)</th>
<th>Vp (m/sec)</th>
<th>Vs (m/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100–1500</td>
<td>1.9</td>
<td>1800</td>
<td>700</td>
</tr>
<tr>
<td>1500–2300</td>
<td>2.2</td>
<td>2800</td>
<td>1500</td>
</tr>
<tr>
<td>&gt;2300</td>
<td>2.5</td>
<td>5600</td>
<td>3000</td>
</tr>
</tbody>
</table>

After Shima et al. (1976).

able geological information such as results of both seismic refraction and explosion surveys, and large aperture array observations of low-frequency microtremors (e.g., Shima et al., 1976; Kushiro Office in Hokkaido Association of Architects and Building Engineers, 1989; Higashi and Kudo, 1992; Miyakoshi and Okada, 1996).

The measurement system used consists of amplifiers, low-pass filters, 16-bit A/D converters, and a notebook computer, all built in a portable case. The acquisition system is connected to the vertical- and three-component velocity sensors with a natural frequency of 1 Hz. The minimum array radii used were 1 m (sites A–C) and 3 m (site D), while the maximum ones were 5 m (site A) and 25 m (sites B–D). The maximum radii were set deliberately to be insufficient for maximum ones were 5 m (site A) and 25 m (sites B–D). The radii used were 1 m (sites A–C) and 3 m (site D), while the sensors with a natural frequency of 1 Hz. The minimum array connected to the vertical- and three-component velocity sensor used in the measurement at the site. The H/V peak frequencies at sites A–D are 4, 2, 0.8, and 0.6 Hz, respectively, but are always higher than the natural frequencies of 0.6 Hz at site A and 0.1–0.2 Hz at sites B–D for the deep Vs structures down to the seismic bedrock (Tables 1–4). At each site, furthermore, the dispersion data shown in Figure 3 is obtained in a frequency range higher than the H/V peak frequency. This suggests that the observed microtremor dispersion curves and H/V spectra, including their peak frequencies, reflect the characteristics of shallow Vs structures above the engineering bedrocks with Vs of about 600–800 m/sec at the sites (Tokimatsu and Miyadera, 1992; Tokimatsu, 1997).

Estimation of Vs Profiles from Microtremor Dispersion and H/V Data

Based on the frequencies of the observed dispersion and H/V data, the following assumptions are made for the proposed joint inversion: (1) the soil profile down to the seismic bedrock at each site consists of a six- to nine-layered half-space, and (2) the deep soil layers below the engineering bedrock have the Vs structures shown in Tables 1–4. This leaves unknown thicknesses and S-wave velocities of the shallow three to five layers to be sought in the inversion. The weighting factors, wR and wH/V, in equation (3) are set as 1 and 0.5, respectively, to make the first and second terms, F_R and F_H/V, in the equation equivalent at the final stage of the inversion. This is based on the empirical fact that the root mean square value, √F, at the final stage of the H/V inversion is, in many cases, about twice that of the dispersion curve inversion (e.g., Tokimatsu and Tamura, 1992; Arai and Tokimatsu, 2004). The conventional inversion using microtremor dispersion data alone was also performed for each site under the same assumptions but with wH/V = 0. For each inversion, about 10–20 or more initial soil layer models were randomly generated, and the iteration analyses were performed using the initial models. Among the resulting solutions, the best one which gives the minimum value of the error ratio criterion √F is selected for the final soil layer model. The variation ranges of the parameters (thickness and S-wave velocity) and the inferred densities and P-wave velocities of the shallow soil layers in the initial models generated at sites A–D are shown in Tables 5–8, respectively. The value of P-wave velocity in each layer is assigned from that of S-wave velocity, provided that the value of Poisson’s
ratio ranges from 0.25 to 0.5 depending on that of S-wave velocity. In this study, the condition numbers of the singular value decompositions in the inversions were always less than 10^5, which are not very large for the condition number, therefore all the inverted solutions have good resolutions.

Figures 3 and 6 show the shallow S-wave velocity profiles estimated from the conventional and proposed joint inversions at sites A–D, respectively. Solid black lines in Figures 3 and 4 are the theoretical dispersion curve of Rayleigh waves and the H/V spectrum of surface waves, respectively, computed for the soil profile estimated from the proposed joint inversion at each site. Broken black line in Figure 3 is the theoretical dispersion curve of Rayleigh waves corresponding to the soil profile from the conventional inversion. With the root mean square values, $\sqrt{F}$, less than 0.1 for the dispersion curves and 0.2 for the H/V spectra, the computed theoretical values show fairly good agreement with the observed ones at all the sites. This suggests that both the conventional and proposed joint inverse analyses have been performed with a reasonable degree of accuracy.

Also shown in Figure 4 in broken light-gray line is the H/V spectrum of surface waves computed for the soil profile from the conventional inversion at each site. The computed H/V spectra, however, do not fit in the observed ones, with the root mean square values over 0.5. At sites A–C, in particular, the peak frequencies of the computed H/V ratios are inconsistent with those of the observed ones. This suggests that the $V_S$ profiles estimated from the conventional inversion could not be reasonably adequate.

Comparing Figures 5 and 6, the $V_S$ profile estimated from the conventional inversion does not coincide with that from the joint inversion at each site. In the figures, the available PS logs at the sites (e.g., Ishihara et al., 1989; Kashima et al., 1994; Matsunaga et al., 1994; Sato et al., 1998) and the standard errors $\sigma_i$ of the parameters $p_j$ evaluated in the inversions (e.g., Wiggins, 1972; Matsu’ura and Hirata, 1982; Horike, 1985; Yuan and Nazarian, 1993; Arai and Tokimatsu, 2004) are also shown as broken black and chained light-gray lines, respectively. In Figure 5, the $V_S$ profile estimated from the conventional inversion using dispersion data alone is inconsistent with the PS log at each site. For example, at sites B and C, the engineering bedrock layer with $V_S$ of about 600–800 m/sec is not identified in the conventional inversion. At sites A and D, the engineering bedrock is identified but the standard error ratios of the related parameters $r_j/p_j$ are extremely large in the conventional inversion, indicating that the estimated results are not reliable. This is because the maximum array radius was set deliberately too small to estimate correctly $V_S$ profile down to the engineering bedrock at each site, thus, the phase velocity data carrying the bedrock information are quite few in the dispersion curve used for the conventional inversion. In Fig-
Figure 4. Comparison of H/V spectra of microtremors (open circles) with those of surface waves (solid black lines) for soil profiles estimated by proposed joint inversions at sites A–D. Also shown in broken light-gray lines are surface-wave H/V spectra computed for the soil profiles estimated by conventional inversions for the sites.

Table 5
Variation Ranges of Thickness and S-Wave Velocity and Inferred Densities and P-Wave Velocities of Shallow Soil Layers in Initial Models for Inversions at Site A

<table>
<thead>
<tr>
<th>Thickness (m)</th>
<th>( \rho ) (t/m³)</th>
<th>( V_p ) (m/sec)</th>
<th>( V_s ) (m/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5–10</td>
<td>1.6</td>
<td>500–1300</td>
<td>50–300</td>
</tr>
<tr>
<td>1–30</td>
<td>1.7</td>
<td>700–1700</td>
<td>100–500</td>
</tr>
<tr>
<td>*</td>
<td>1.9</td>
<td>1500–2700</td>
<td>400–1200</td>
</tr>
</tbody>
</table>

*The bottom of this layer connects to the top of the deep soil layers at site A (Table 1).

Table 6
Variation Ranges of Thickness and S-Wave Velocity and Inferred Densities and P-Wave Velocities of Shallow Soil Layers in Initial Models for Inversions at Site B

<table>
<thead>
<tr>
<th>Thickness (m)</th>
<th>( \rho ) (t/m³)</th>
<th>( V_p ) (m/sec)</th>
<th>( V_s ) (m/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5–20</td>
<td>1.7</td>
<td>350–1500</td>
<td>25–400</td>
</tr>
<tr>
<td>1–20</td>
<td>1.4</td>
<td>350–1500</td>
<td>25–400</td>
</tr>
<tr>
<td>1–40</td>
<td>1.5</td>
<td>350–1500</td>
<td>25–400</td>
</tr>
<tr>
<td>10–100</td>
<td>1.8</td>
<td>700–2200</td>
<td>100–800</td>
</tr>
<tr>
<td>*</td>
<td>2.0</td>
<td>1700–2400</td>
<td>500–1000</td>
</tr>
</tbody>
</table>

*The bottom of this layer connects to the top of the deep soil layers at site B (Table 2).

Table 7
Variation Ranges of Thickness and S-Wave Velocity and Inferred Densities and P-Wave Velocities of Shallow Soil Layers in Initial Models for Inversions at Site C

<table>
<thead>
<tr>
<th>Thickness (m)</th>
<th>( \rho ) (t/m³)</th>
<th>( V_p ) (m/sec)</th>
<th>( V_s ) (m/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5–20</td>
<td>1.6</td>
<td>500–1700</td>
<td>50–500</td>
</tr>
<tr>
<td>0.5–20</td>
<td>1.7</td>
<td>500–1700</td>
<td>50–500</td>
</tr>
<tr>
<td>1–50</td>
<td>1.8</td>
<td>500–1700</td>
<td>50–500</td>
</tr>
<tr>
<td>2–100</td>
<td>1.8</td>
<td>700–2200</td>
<td>100–800</td>
</tr>
<tr>
<td>*</td>
<td>2.0</td>
<td>1500–2700</td>
<td>400–1200</td>
</tr>
</tbody>
</table>

*The bottom of this layer connects to the top of the deep soil layers at site C (Table 3).

Table 8
Variation Ranges of Thickness and S-Wave Velocity and Inferred Densities and P-Wave Velocities of Shallow Soil Layers in Initial Models for Inversions at Site D

<table>
<thead>
<tr>
<th>Thickness (m)</th>
<th>( \rho ) (t/m³)</th>
<th>( V_p ) (m/sec)</th>
<th>( V_s ) (m/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1–40</td>
<td>1.8</td>
<td>500–1500</td>
<td>50–400</td>
</tr>
<tr>
<td>1–40</td>
<td>1.8</td>
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<td>2–80</td>
<td>1.8</td>
<td>700–2000</td>
<td>100–700</td>
</tr>
<tr>
<td>*</td>
<td>1.9</td>
<td>1500–2400</td>
<td>400–1000</td>
</tr>
</tbody>
</table>

*The bottom of this layer connects to the top of the deep soil layers at site D (Table 4).
Figure 5. Comparison of shallow S-wave velocity profiles estimated by conventional inversion using dispersion curves of microtremor vertical motions (solid black lines) with available PS logs (broken black lines) (e.g., Ishihara et al., 1989; Kashima et al., 1994; Matsunaga et al., 1994; Sato et al., 1998) at sites A–D. Also shown in chained light-gray lines are standard errors of soil layer models evaluated by conventional inversion.

Figure 6 shows that the difference between the $V_S$ profile from the microtremor method and that from the $PS$ log (borehole method) still exists at each site. This could be partly due to the limitation of capability of the microtremor method for estimating $V_S$ profiles. It is naturally suggested that the reliability and accuracy of $V_S$ profiles estimated from...
the microtremor method without borehole could be equal to or inferior to those from the borehole method. However, it has also been indicated that the local site effects (one-dimensional site amplification factors for vertically propagating $S$ waves) computed using sedimentary $V_s$ profiles estimated from the microtremor method are consistent with those from earthquake ground motions recorded with downhole arrays of seismometers installed in the sediments at several sites (e.g., Tokimatsu, 1997; Arai and Tokimatsu, 1998). This suggests that the differences between $V_s$ profiles from the two different methods as shown in Figure 6 could be permissible for evaluating local site effects during earthquakes.
Sensitivity Analysis of Dispersion Curve and H/V Spectrum

To investigate the reason why the resulting \( V_S \) profiles estimated from the proposed joint inversion, particularly the profile of the engineering bedrock, are improved with respect to those from the conventional inversion, sensitivity analyses are conducted for the soil layer models derived from the joint inversions at sites A–D. The absolute value of the non-dimensional partial derivative, that is, sensitivity, of Rayleigh-wave phase velocity, \( c_R(f) \) at a frequency \( f \), for any of the parameters in the \( j \)th layer of the soil model, \( P_{ji} = (H, P, V_P, V_S) \), can be expressed as (Horike, 1985)

\[
D_{ji}^S(f) = \left| \frac{\partial c_R(f)}{\partial P} \right| \bigg|_{P = P_j} .
\]

(6)

Similarly, the sensitivity of the surface-wave H/V ratio, \( (H/V)_{S_j}(f) \) at a frequency \( f \), for any of the parameters \( P_{ji} \) can also be expressed as (Arai and Tokimatsu, 2004)

\[
D_{ji}^{HV}(f) = \left| \frac{\partial (H/V)_{S_j}(f)}{\partial P} \right| \bigg|_{P = P_j} .
\]

(7)

The larger the values of \( D_{ji}^S \) and \( D_{ji}^{HV} \), the more sensitive \( c_R \) and \( (H/V)_{S_j} \) are to the parameter \( P_{ji} \), respectively.

Figure 7a–d shows the variations of the values of \( D_{ji}^S \) (sensitivities) of Rayleigh-wave phase velocity and surface-wave H/V ratio with frequency with respect to the \( S \)-wave velocity of the top four layers including the engineering bedrock, \( D_{1}^S(f) \) and \( D_{4}^S(f) \), respectively, at site D. Also shown in Figure 7e–g are those with respect to the thickness of the top three layers above the bedrock, \( D_{2}^S(f) \) and \( D_{3}^S(f) \), respectively, at site D.

In the frequency range greater than 1 Hz, the values of \( D_{ij}^S(f) \) are equal to or at most 10 times those of \( D_{ij}^S(f) \), respectively (Fig. 7a–c, e, f). However, the values of \( D_{ij}^{HV}(f) \) and \( D_{ij}^{HV}(f) \), which are related strongly to the bedrock \( V_S \) structure, are about 10–100 times those of \( D_{ij}^{HV}(f) \) and \( D_{ij}^{HV}(f) \), respectively (Fig. 7d, g).

In the frequency range lower than 1 Hz, at which no phase-velocity data is obtained but the H/V spectrum has a significant peak, the values of \( D_{ij}^{HV}(f) \) and \( D_{ij}^{HV}(f) \) are almost equal to those in the frequency range greater than 1 Hz. Similar trends exist in the different soil layer models at sites A–C. This indicates that the H/V spectrum of surface waves is more sensitive to the parameters reflecting the bedrock \( V_S \) structure than the phase velocity of Rayleigh waves for many realistic soil layer models with a distinct H/V peak, although the sensitivities \( D \) of soil layer models vary during the inversion process. Therefore, the bedrock \( V_S \) profiles could be estimated more reliably by using microtremor H/V data in addition to dispersion data. Thus, it is finally concluded that the proposed joint inversion of both dispersion curve and H/V spectrum of microtremors is promising for estimating the \( V_S \) profile down to the engineering bedrock, even though the frequency range of the observed microtremor dispersion data is insufficient to estimate the whole \( V_S \) profile.

Detailed examination of Figure 7 further indicates that the values of sensitivity \( D \) at site D vary in a complex way with respect to frequency. Similar complexity also exists at the other sites. From the theoretical formulations of the Rayleigh-wave phase velocities and the surface-wave H/V ratios considering the effects of the fundamental- and higher-modes (Tokimatsu et al., 1992; Arai and Tokimatsu, 2004), the response functions of Rayleigh and Love waves, \( (A_R/k_R)(f) \) and \( (A_L/k_L)(f) \) (e.g., Regan and Harkrider, 1989; Hisada et al., 1991), up to fourth higher mode are shown in Figure 8a and b, respectively. With the figures and the theoretical formulations, it is suggested that the contribution ratio of each mode to the theoretical dispersion curve and the H/V spectrum could change drastically depending on frequency. Therefore, the complexity of the mode contributions with respect to frequency could affect the shapes of the sensitivity spectra in Figure 7. It is also suggested that the non-dimensional partial derivatives of the response functions for the Rayleigh- and Love-wave modes with respect to the soil layer model parameters \( P_i \), \( [P(A_R/k_R)(f)](\partial(A_R/k_R)(f))/\partial P \) and \( [P(A_L/k_L)(f)](\partial(A_L/k_L)(f))/\partial P \), respectively, might have influenced the sensitivities \( D \). Currently, more detailed discussion on this matter appears difficult and requires further research because the problems are very complicated.

Despite their frequency-dependent nature and uncertainty, the values of the parameters in equation (3) (\( R/L, w_k \), and \( w_H V \)) are assumed to be constants (0.7, 1, and 0.5, respectively) in the frequency range considered, based on the previous studies (e.g., Matsushima and Okada, 1990b; Tokimatsu and Tamura, 1992; Arai and Tokimatsu, 2000, 2004). This assumption appears to have insignificant effects on the final solutions because the inverted results are reasonable at all the four sites with different soil conditions. In addition, those three values are currently unable to be determined and should be investigated further from such as the sensitivity analyses of the parameters.

Conclusions

This article has introduced a methodology for estimating the S-wave velocity profile of subsurface soils using both microtremor dispersion curve and H/V spectrum. The results of the study are summarized as follows:

1. A joint inverse analysis using both microtremor dispersion curve and H/V spectrum is presented for estimating \( V_S \) profiles of subsurface soils. In the inversion, both microtremor dispersion and H/V data are assumed to be the
Figure 7. Absolute values of nondimensional partial derivatives (sensitivities) for Rayleigh-wave phase velocity (solid line) and surface-wave H/V ratio (broken line) with respect to (a)–(d) S-wave velocity of top four layers including engineering bedrock and (e)–(g) thickness of top three layers above the bedrock in soil layer model inverted for site D, Tokyo, Japan.
Rayleigh-wave dispersion curve and the surface-wave (both Rayleigh and Love) H/V spectrum that have been theoretically derived by taking into account the effects of their fundamental and higher modes.

2. The proposed joint inversion as well as the conventional one using only dispersion data is performed at four sites where shallow VS profiles down to engineering bedrock are available. The VS profiles estimated by the proposed joint inversion are more consistent with available downhole velocity logs than those by the conventional method. In particular, the proposed inversion shows significant improvement in estimating bedrock VS structures compared to conventional inversion.

3. Sensitivity analyses indicate that the surface-wave H/V ratio is sensitive to the bedrock VS structure more than the Rayleigh-wave phase velocity, confirming that the proposed joint inversion including H/V spectrum is promising.

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References


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