Array Observations of Microseismic Noise and the Nature of H/V in the Mississippi Embayment

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Abstract Ambient ground-motion data were collected using phased seismic arrays in fall 2002 and spring 2007 within the Mississippi embayment and at a single station external to the embayment. These data allowed us to determine the wave-field composition of ambient noise for understanding wave-propagation mechanisms giving rise to spectral peaks using Nakamura’s H/V technique. Ambient ground motions in the frequency band of 0.1–0.33 Hz (10–3 sec period) were dominated by spatially localized Rayleigh- and Love-wave microseisms generated by high-ocean waves along the North American seaboard in the time periods of analysis. Seismic waves important in forming the H/V peak near 4 sec period are composed of relatively high-phase velocity Rayleigh and Love waves that convert to propagating homogeneous shear waves in the thick unconsolidated sediments of the embayment. The H/V resonant period is controlled by both constructive and destructive interference of these shear waves. A simple relationship for the H/V peak is given using a propagator matrix formulation that predicts the resonance frequency of a layered medium for surface wave motion at the base of the system. The amplitude of the observed H/V peak, however, does not give an accurate estimate of shear-wave amplification because it depends on the slowness of the incident wave. The inconsistency in estimated average shear-wave velocities using the H/V method and differential travel times of local earthquake Sphases in the Mississippi embayment may be explained by misidentification of Sph-wave conversion points from deeper interfaces.

Introduction

A variety of field studies have shown that Nakamura’s (Nakamura, 1989) empirical technique for estimating shear-wave site resonance frequencies is a robust method that can yield useful information about the structure of a site in the near surface that may be used for shaking hazards assessments (Lermo and Chávez-García, 1993; Field et al., 1994; Lermo and Chávez-García, 1994b; Field and Jacob, 1995; Theodulidis and Bard, 1995; Malagnini et al., 1996; Bour et al., 1998; Haghshenas et al., 2008; Lozano et al., 2008). The method employs three-component recording of ambient ground motions for a site of interest and then forms the ratio of the amplitude or power spectra of horizontal component ground motions (H) to the vertical component ground motions (V). The H/V ratio usually displays a set of peaks in ground motion that are interpreted in terms of the resonance of vertically incident shear waves in a simple-layered velocity structure. The method is purely empirical and based on a set of intuitive assumptions about the wave propagation involved in ambient ground motions. In particular, the H/V ratio is interpreted in terms of a simple spectral transfer function where equal amplitude horizontal and vertical wave motions at depth are separately affected by the intervening structure on their way to the free surface. The major assumption is that shear waves are horizontally polarized and will be amplified by the usual decrease in shear-wave impedance in the near surface and will also resonate within the layered structure. The amplitude of the vertical wave field is assumed to be unaffected.

Theoretical studies using distributions of surface noise sources have shown that heterogeneous ambient noise fields do yield synthetic results for plane-layered velocity models consistent with Nakamura’s assumptions (Field and Jacob, 1993; Lachet and Bard, 1994) and consistent with the propagation of Rayleigh- and Love-surface waves (Konno and Ohmachi, 1998). Nakamura (1989) also checked his basic assumption on the nature of vertical motions using borehole seismic observations. The frequencies of spectral peaks in the H/V ratio seems to be a robust feature that can be related to site structure, but it is less clear if the amplitude of the peak is a useful measure of absolute ground-motion amplification (Lachet and Bard, 1994).
The purpose of this study is to understand the detailed wave propagation mechanisms involved at the heart of Nakamura’s technique and extend the results that Bodin and Horton (1999) and Bodin et al. (2001) obtained for Earth structure in the Mississippi embayment (Fig. 1). The question can be asked whether there is some bias in Nakamura’s method that can lead to a bias in the estimated average velocity using simple interpretations of the resonant period peak. We are also interested in the question of whether the amplitude of the H/V spectral peak is a quantitative estimate of spectral amplification for earthquake shear waves in the embayment. The answer to both questions will have important implications for the study of Earth structure in the unconsolidated sediments and the estimation of shaking hazards from large earthquakes in the New Madrid seismic zone (NMSZ).

The Mississippi embayment (Fig. 1) overlies the NMSZ in the central United States and consists of a thick succession of unconsolidated coastal plain sediments arranged in a shallow synformal structure (Stearns, 1957; Stearns and Marcher, 1962). The average shear-wave velocity of the sediments is approximately 700 m/sec (Langston, 2003a), and thicknesses of up to a kilometer in the area predict low-frequency site resonance effects. Bodin and Horton (1999) and Bodin et al. (2001) showed that H/V peak periods varied from about 3 to 5 sec and that period correlated with sediment thickness.

Using constraints on sediment thickness from well log and reflection seismology depth estimates (Dart and Swolfs, 1998) they also found that the average shear-wave velocity inferred from the resonant period increased with sediment thickness throughout the embayment (Fig. 2). However, they pointed out that the average velocity estimates obtained from H/V measurement interpretation were significantly higher than that obtained from the interpretation of S – Sp differential travel times (Chen et al., 1996). The Sp phase is a common observation on local earthquake seismograms in the NMSZ and, because it is so large, has been attributed to the large velocity contrast between the unconsolidated sediments and the underlying high-velocity Paleozoic limestones (Andrews et al., 1985; Chen et al., 1996; Langston, 2003b). The discrepancy between average shear-wave velocities determined from the H/V and Sp data is unexplained and might suggest significant biases inherent in either method. For example, Nakamura’s technique is empirical and the observed spectral peaks are unusually low frequency for this kind of study.

Nakamura’s method relies on a single receiver measurement and makes assumptions about the ambient ground-motion noise field. To answer our questions, we need to

Figure 1. Index map showing political boundaries, contours of unconsolidated sediment thickness (in meters) for the Mississippi embayment (after Bodin et al., 2001), regional seismicity (dots), locations of the array experiments (circled stars), location of the Wilson Well site (triangle), and PARM and WVT seismic stations.
understand the nature of the measurement. Specifically, what is the composition of the ambient noise field in the period band of 3–5 sec? What waves are present? Are they coherent? If they are coherent, what are wave propagation directions and horizontal phase velocities? In other words, what is the source of the ambient ground motion and how do these waves propagate? Once the wave propagation characteristics of the noise field are known, it may become possible to understand how those waves interact with the structure of the embayment. In other words, we are interested in controlling parameters of what has previously been an uncontrolled, empirical seismic experiment. Although some inferences on the truth of a method can be made by comparing results of several methods and site analysis (e.g., see review by Haghshenas et al., 2008), it is important to understand the actual wave propagation mechanisms involved in any experiment to understand method limitations.

Our approach is to use broadband phased arrays of seismometers to record ambient ground motions within the embayment. We use the array observations to perform a wave-field decomposition to determine horizontal phase velocities and azimuths of the ambient noise to determine the source of the noise and wave types involved. The array data are processed using standard frequency–wavenumber beamforming techniques (e.g., Nawab et al., 1985) and, for one array, using newly developed techniques of wave gradio-

Figure 2. Estimated shear-wave velocity versus sediment thickness from interpretation of H/V spectral peaks (triangles) from Bodin et al. (2001) and S – S_p travel times (crosses) from Chen et al. (1996). The S – S_p travel-time data were corrected for using an average P-wave velocity of 2.1 km/sec rather than Chen et al.’s (1996) value of 1.8 km/sec based on a study of velocity structure by Langston (2003a). This serves to bring the H/V and S_p data results somewhat closer together. However, the two fields of points are still clearly separated and show an unexplained inconsistency. The stars show average velocities inferred from Mooring array data (at 700 m sediment thickness) and Marked Tree array data (at 800 m sediment thickness). Error bars for these estimates are based on a conservative estimate of the half-width of the H/V peak as ±0.02 Hz.

Array Observations of Microseismic Noise and the Nature of H/V in the Mississippi Embayment

Two temporary seismic arrays were deployed within the Mississippi embayment, one near Marked Tree, Arkansas, and the other near the small village of Mooring, Tennessee (Fig. 1). These sites were chosen to be near the locations of the two large explosion sources associated with the embayment seismic excitation experiment (Langston et al., 2006) because other geotechnical and seismological studies have been performed to investigate the velocity structure of the sediments at each site. The Mooring array (Fig. 3) was deployed in 2002 immediately after the experiment. At that time, we postulated that the ambient noise field at 3–4 sec period consisted of very slowly propagating (~1–2 km/sec) Rayleigh and Love waves within the thick, slow embayment sediments. The array was designed with a small aperture of only ~2 km to sample about one-half wavelength for waves with the expected low-phase velocities. However, we found at the time that ambient noise signals were very coherent across this array and had relatively high-phase velocities of 3–4 km/sec. Unfortunately, the resolution of the array for these high-phase velocities is poor (Fig. 4) using standard frequency–wavenumber spectral methods (Nawab et al., 1985).

The Mooring array consisted of eight Program for Array Seismic Studies of the Continental Lithosphere (PASSCAL) three-component, broadband Guralp CMG-40T instruments with Reftek seismic recorders and a center element with a Guralp CMG-3ESP that was available at the Center for Earthquake Research and Information (CERI). The center element suffered vandalism by an animal early in the night of 26 November when the array was deployed. We recorded about 12 hr of data before removing the array.

The Marked Tree array (Figs. 1 and 3) was deployed 22–24 March 2007. It consisted of six CMG-6TD three-component, 24 bit systems that are part of the aftershock deployment inventory of CERI. The array configuration comes from an innovative design suggested by Fred Fowllowill (F. Followill, personal comm., 2006). As Figures 3 and 4 show, the increased aperture to ~6 km dramatically improves the frequency–wavenumber resolution for high-velocity, 4–5 sec surface waves. There are no significant side lobes in the frequency–wavenumber response for waves in the frequency bands shown in Figure 4 for slownesses up to 1 sec/km.

The thickness of unconsolidated sediments under the Marked Tree array is 850 m and under the Mooring array is 700 m, using the compilation of data from Dart (1992) and Dart and Swolfs (1998) (Fig. 1).
Data Analysis

H/V Ratios

Vertical and horizontal component data from each array and from station WVT for a 1 hr-time period were processed to obtain the H/V spectral ratio. The array data were chosen to minimize cultural noise generated by tractors from adjacent farm fields and traffic from nearby roads. Data for the hour of 08:00 coordinated universal time (UTC) in the early morning of 26 November 2002 were processed for the Mooring array and WVT, and data for the hour of 06:00 UTC on 24 March 2007 were processed for Marked Tree and WVT. We were interested in knowing the composition of the ambient noise field for a relatively short period of time for this study. Because the Marked Tree array was only deployed for 2 days and Mooring for 12 hours, we will not address the issue of time variations in the ambient noise field. This issue needs a longer array deployment to collect appropriate data.

The data were corrected for instrument response, the three components of ground motion were windowed, and a fast Fourier transform was used to compute amplitude spectra. Amplitude spectra from all vertical channels and all horizontal channels at each array were summed and the average taken. The spectra were smoothed using a 20-point averaging operator and then the ratio of H/V taken. The data at WVT were treated the same way in that the amplitude spectra for the two horizontal components were averaged before the smoothing operator was applied. We did not use a pseudovector addition of the horizontal amplitude spectra by taking the square root of the sum of squares of the two horizontal components as is sometimes done in H/V studies. The amplitude spectra of all horizontal components were similar in magnitude. Averaging all array elements had a tendency to naturally smooth the amplitude spectra, and the net effect was only to slightly reduce H amplitude levels over the pseudovector addition method.

Results are shown in Figure 5. At Marked Tree, there is a prominent H/V peak between 0.2 and 0.25 Hz (4–5 sec period) with an amplitude of close to 12. At Mooring, the H/V peak is similar but is centered at 0.25 Hz (4 sec period) and attains an amplitude of approximately 8. WVT (Waverly, Tennessee) is located outside the Mississippi embayment and is installed in high-velocity Paleozoic limestones. The H and V spectra are remarkably similar and show the classic microseism peak near 0.15–0.2 Hz (5–7 sec period). The H/V ratio in the band of 0.1–0.4 Hz shows no obvious peak structure and is close to unity within 50%. We take the WVT observations as an indication of the H and V spectra of ambient noise before these waves interact with deep sediments of the embayment. In that sense, the ambient noise has characteristics consistent with Nakamura’s assumptions where H and V motions are essentially the same amplitude before interacting with near-surface, low-velocity sediments. The observed H/V peak frequencies are consistent with signals observed in Bodin and Horton (1999) and Bodin et al. (2001).

Array Analysis

To learn about the source of the ambient wave field recorded by the Marked Tree array, we performed narrowband, frequency–wavenumber analysis for 1 hr of data. Figure 6 shows the result in a band that includes the H/V peak (0.2–0.25 Hz), and the band just to side from 0.3 to 0.4 Hz. Between 0.2 and 0.25 Hz, the ambient ground-motion wave field is exceptionally coherent and shows a well-defined direction of north-northeast–northwest with waves traveling as fast as 4.3 km/sec on the vertical components to 3.1 km/sec on the horizontal components. The beam pattern is close to the ideal plane wave beam pattern as shown in Figure 4 and suggests an isolated geographic source to the northeast for ambient noise waves. Particle motion analysis in this direction for vertical and radial band-passed data show flattened elliptical motions with...
retrograde orbits indicative of Rayleigh waves. Frequency–wavenumber analysis of the left-hand side of the H/V peak between 0.15 and 0.2 Hz (not shown) gives a very similar view except the beam pattern is somewhat wider because of the decrease in frequency.

The higher frequency band pass (Fig. 6) shows that waves are less coherent, over this hour’s data and seem to primarily arrive from more easterly azimuths. The beam patterns are stretched from northern to eastern azimuths and are distorted from the ideal beam pattern for this frequency band as shown in Figure 4. The beam pattern for the north–south component has two significant peaks, one showing waves that have phase velocities of 4–5 km/sec and the other for waves of roughly 1 km/sec. The low-phase velocity values suggest waves are trapped in the low-velocity sediments of the embayment and possibly come from local sources to the east. The higher phase velocity waves arriving from various directions in the northeast quadrant are likely to be Sn and Lg waves that are trapped in the uppermost mantle and crust, respectively. These kinds of waves can be considered to be higher mode surface waves but do have clear raylike characteristics (Langston et al., 2002).

Figure 4. Theoretical narrowband plane wave responses for the Marked Tree (top row) and Mooring (bottom row) arrays. The left-hand panels show frequency–wavenumber array responses for an incident plane wave from 45° azimuth traveling at 3 km/sec for the passband of 0.2–0.25 Hz. The right-hand panels show the response for a plane wave traveling from the east with a velocity of 3 km/sec in the passband of 0.3–0.4 Hz. Beam patterns were computed using a sampling interval of 0.01 sec/km in horizontal slowness. The extremely broad peak responses (e.g., large red areas representing 90% of the frequency–wavenumber spectral energy) for the Mooring array show that the array was constructed with too small of an aperture to adequately resolve the speed and azimuth of surface waves in the frequency bands of interest. The Marked Tree array, however, shows very good resolution in these frequency bands with no anomalous side peaks out to 1 sec/km slowness.
Thus, the Marked Tree array analysis shows that the dominant component of waves that make up the H/V peak are primarily 4–5 sec period Rayleigh and Love waves traveling at 3.1 km/sec from a geographically isolated source to the northeast. Higher frequency ambient noise to the side of the H/V peak arrives from more easterly azimuths and appears to have several source areas, some of which may be local. The higher frequency wave field shows a combination of high-phase velocity waves that come from a great distance and low-phase velocity waves that must locally couple into the embayment sediments.

Although the Mooring array was underdesigned for analyzing long-period, high-velocity surface waves using beam forming techniques, it can be utilized as a gradiometer for obtaining the same information. Wave gradiometry is a relatively new method that uses the spatial gradients of the wave field to determine azimuth and slowness of incident seismic waves that pass through arrays whose apertures are a fraction of the wavelength under consideration. Seismic displacements of the array are differenced to form the spatial derivatives of the wave field using a Taylor series expansion about some reference station. As in beam forming, a wave model is assumed to process the displacement gradients to obtain horizontal slowness and azimuth of propagation. Gradiometry also yields changes in geometrical spreading and radiation pattern of the wave. The wave model in gradiometry assumes a point source in cylindrical coordinates in which waves travel with distance-dependent horizontal slowness and have arbitrary geometrical spreading and azimuthal radiation patterns. Details of the technique can be found in Langston (2007a,b,c) and Langston and Liang (2008).

The wavelengths of 4 sec, 3–4 km/sec surface waves are 12–16 km. The aperture of the Mooring array is 1–2 km depending on which stations are included. Thus, the array is about 10% of the expected wavelength and can be used to form finite differences of the seismic observations to obtain spatial wave gradients. First we derived azimuth from filtered horizontal components assuming the isotropic point source approximation by computing displacement.

Figure 5. Computed H amplitude spectra, V amplitude spectra, and H/V ratios for the passband between 0 and 1 Hz for data recorded at the arrays and simultaneously at the WVT station. One hour of data were used to compute the spectra. Data for the Marked Tree array were from 06:00 UTC on 24 March 2007 and for the Mooring array from 08:00 UTC on 26 November 2002. The H/V ratios have been multiplied by factors shown in the plots to enable comparison with the original H and V spectra. It should be understood that the H/V spectra have no units even though we are using the spectral amplitude scale in each plot.
Figure 6. Narrowband frequency–wavenumber spectra for 0.2–0.25 Hz and 0.3–0.4 Hz data recorded by the Marked Tree array. The top, middle, and bottom panels show spectra computed from vertical, east–west, and north–south station components, respectively. The peak values shown by a white × on each plot are chosen for wave horizontal slowness and azimuth. This is a good estimate for the 0.2–0.25 Hz band because the spectral responses are very close to the ideal array plane wave response shown in Figure 4. The higher frequency band shows more complex features implying several sources for incident waves.
gradients of the horizontal components and finding the azimuth of wave propagation, $\theta$, using (Langston and Liang, 2008)

$$\tan \theta = \frac{\partial u}{\partial x} = \frac{\partial v}{\partial y},$$

(1)

where $u$ and $v$ are the east–west and north–south horizontal motions, respectively. Results for the 0.2–0.3 Hz passband that includes the observed H/V peak are shown at the top of Figure 7. The rose diagrams show that horizontal ambient ground motions in the early morning of 26 November 2002, traveled in north-northeast–northeast or south-southwest–southwest directions. Directionality and speed

![Figure 7](image-url)
can be resolved by performing a gradiometric analysis of the radial components of motion resolved in this azimuth using the time domain method (Langston, 2007b) over the same passband. These results are shown at the bottom of Figure 7. Large wave packets during a 200 sec time window are seen to propagate in a direction of 200–250° azimuth with horizontal speeds from 3 to 5 km/sec. Thus, waves that compose the H/V peak at Mooring are also composed of relatively high-velocity surface waves moving to the southwest.

Ambient Ground-Motion Sources

The H/V peak in the Mississippi embayment occurs very near the maximum of the 5 sec global microseism peak (Peterson, 1993), and the array and gradiometry analyses show that the waves in this band are relatively coherent and seem to come from specific source areas. Microseisms in this band are generated by linear and nonlinear loading of the continental shelves from storm surf (Longuet-Higgins, 1950; Friedrich et al., 1998; Bromirski et al., 1999; Gerstoft et al., 2006). Like Gerstoft et al. (2006) we utilized a National Oceanic and Atmospheric Administration (NOAA) hindcast ocean wave model (Tolman, 2005) to investigate possible source areas for the microseisms.

Figure 8 shows a rendering of the wave height model during the analysis time periods when the arrays and WVT station recorded data. Microseisms recorded by the Mooring array apparently were generated by high waves off of northeastern Newfoundland from a storm in the North Atlantic because the area of high-wave action correlates well with the azimuths obtained by the three-component gradiometry results. The same pattern occurs for the source of microseisms recorded by the Marked Tree array in April 2007. Notably, there is no evidence in the array data for microseism sources from the eastern Pacific Ocean. It appears that 3–5 sec period surface waves do not propagate as efficiently across the tectonic and mountainous zones of western North America compared to the continental shield areas of eastern North America.

Comparison of Ambient Ground Motions In and Outside the Embayment

The array analysis and wave height models show that waves that compose the observed H/V peak are surface waves generated by storm surf along the northeastern continental shelf of North America. The H and V spectra for the WVT station outside the embayment show equal ground motions and have no amplification effects due to near-surface structure because the station sits on high-velocity limestones. In the spirit of Nakamura’s method, we can assume that WVT station ambient ground motion represents a measure of the input motion that occurs beneath stations of the embayment. Taking WVT as a type of hard rock reference station, we formed spectral ratios of horizontal and vertical motions between the arrays and WVT to investigate how H and V spectra change within the embayment.

Figure 9 displays the result. The ratio of array-H to WVT-H is very close to the array H/V ratios. H is amplified by a factor of 8 and 12 at Marked Tree and Mooring, respectively, compared to WVT. The frequency at each peak is also close to that seen in the array H/V ratios although peaks appear broader. The ratios of vertical spectra between arrays and WVT are not flat, however. The ratios of array-V to WVT-V both start to increase near the peak of H motion and continue to higher frequency. At Marked Tree, this was the portion of the

Figure 8. Ocean wave heights from the WAVEWATCH III hindcast models published by the NOAA (Tolman, 2005) for the data time periods used from the Marked Tree and Mooring array experiments. The solid white lines show the general azimuth directions inferred from the width of the frequency–wavenumber array response and gradiometry analyses for the frequency band 0.2–0.25 Hz. The dotted lines are directions to high-wave sources along the western margin of North America. The northeastern sources are only seen in the array data. Observed seismic waves were generated by storm-generated surf along the coasts of northeastern North America. Note that the important parts of the wave height model pertaining to the generation of ambient seismic noise are concerned with waves near the coasts. High waves offshore, such as for the 2002 time period, are not effective in generating seismic noise.
spectrum that started to show the effects of local sources of low-velocity noise and off-azimuth, high-phase velocity microseisms. Both vertical spectral ratios also show interesting peaks between 0.5 and 0.6 Hz that are actually spectral regions where the H and V spectra have the same amplitude. At frequencies higher than 0.6 Hz, the H and V spectra diverge. These station spectral ratios suggest that the effect of the embayment is greatest for horizontal component microseisms; amplification of horizontal motions gives rise to the first order effect of the H/V spectral peak. But, vertical motions are not immune from spectral changes. The array data and the array/station spectral ratios suggest that embayment

Figure 9. H and V spectral ratios of array data to data from WVT station.
structure and local noise sources, within the embayment, contribute to increasing signal levels with frequency. Thus, Nakamura’s assumption of unchanging vertical motions is not strictly true within the embayment and is an important effect near the microseism peak.

Wave Propagation Models for H/V Ratios

These observations and clues can now be used to understand the basic mechanism of amplification that gives rise to the H/V peak. Figure 10 is a velocity model derived from acoustic well logs of two adjacent wells drilled in the unconsolidated sediments (the Wilson 2-14 well) and a well that penetrated through to the pre-Cambrian basement (Dow Chemical Wilson #1 well). The unconsolidated sediments form an extreme low-velocity zone atop a thick section of Paleozoic limestones and dolomites. There are small velocity inversions near the bottom of the Paleozoic section due to Cambrian clastic rocks of the Reelfoot rift. The array and inversions near the bottom of the Paleozoic section due to Paleozoic limestones and dolomites. There are small velocity penetrations through to the pre-Cambrian basement (Dow consolidated sediments (the Wilson 2-14 well) and a well that acoustic well logs of two adjacent wells drilled in the unconsolidated sediments (the Wilson 2-14 well) and a well that penetrated through to the pre-Cambrian basement (Dow Chemical Wilson #1 well). The unconsolidated sediments form an extreme low-velocity zone atop a thick section of Paleozoic limestones and dolomites. There are small velocity inversions near the bottom of the Paleozoic section due to Cambrian clastic rocks of the Reelfoot rift. The array and inversions near the bottom of the Paleozoic section due to Paleozoic limestones and dolomites. There are small velocity

\[ T_n = \frac{4H}{n V_S} \left( 1 - \frac{V_S^2}{c^2} \right)^{1/2}, \quad n = 1, 3, 5, \ldots \]  

(2)

where \( T_n \) is the period for constructive interference, \( H \) the layer thickness, \( V_S \) the shear-wave velocity of the layer, \( c \) the horizontal phase velocity, and \( n \) the number of interfering plane waves, or harmonics, between the bottom and top of the layer. Setting \( n = 1 \) and \( c = \infty \) gives the usual equation associated with interpretations of H/V peaks in Nakamura’s method:

\[ T = \frac{4H}{V_S}. \]  

(3)

Using estimates of shear-wave velocity of the sediments and horizontal phase velocity of the waves determined here, there is only a 2–3% difference between values from equation (2) and the simplified equation (3).

However, the observed wave field does not consist of plane \( SH \) waves with near-vertical incidence but are actually surface waves that are inhomogeneous because they decay exponentially away from the free surface and have no component of vertical wave propagation. Even so, the usual conditions of wave propagation apply in that the boundary condition at a horizontal, welded boundary requires continuity of horizontal phase in addition to continuity of displacement and stress (resolved on the boundary, e.g., Ben-Menahem and Singh, 1981). Continuity of horizontal phase requires that horizontal phase velocity must match across the boundary and is simply another statement of Snell’s law. The resonance represented by equation (2) requires a positive reflection coefficient at the free surface and a negative, real-valued reflection coefficient at the base of the layer. The interaction of inhomogeneous waves with a layer gives rise, by definition, to postcritical \( P \)- and \( S \)-propagating waves that reverberate within the layer, if the layer wave velocities are low enough. By postcritical, we mean that the resulting \( P \) and \( S \) waves will have angles of incidence greater than \( P \)- and \( S \)-wave critical angles at the base of the layer. Reflection coefficients will be complex valued, and the conditions for constructive interference different than that shown in equation (2).

To investigate the influence of inhomogeneous waves interacting with a layered structure, we first compute radial and transverse responses assuming incident inhomogeneous plane waves from below. Figure 11 shows the results of a number of plane wave calculations for incident \( SH \) and \( SV \) waves under an Earth model that contains 1001 thin layers to approximate the unconsolidated sediments (above 850 m seen in Fig. 10) over a simple half-space with \( V_P = 6.0 \) and \( V_S = 3.5 \) km/sec. \( Q_P \) and \( Q_S \) of 200 and 100, respectively, were assumed for all layers in the model, consistent with results from high-frequency surface wave propagation in the embayment (Langston et al., 2005). Horizontal component plane wave responses were computed using a propagator matrix method described in Langston (2003a). \( SH \) and \( SV \) amplitude spectra were computed for the

Figure 10. A \( P \)- and \( S \)-wave velocity model derived from acoustic well log data from the Wilson 2-14 and Dow Chemical Wilson #1 well. Rock types were obtained from the geological logs. A sediment model and travel times of body-wave reflections and conversions were used to constrain the shear-wave model for the unconsolidated sediments (Langston, 2003a). \( V_S \) was derived from \( V_P \) in the deeper Paleozoic section by simply assuming a Poisson solid.
embayment model and the response for a half-space model without the embayment sediment model. The spectral ratio of the embayment model to the half-space model was taken to examine the direct amplification effect of embayment sediments. As Konno and Ohmachi (1998) point out, spectral ratios of this sort often contain infinite amplitude peaks because of complete constructive interference at Rayleigh- or Love-wave resonance. We avoid these artificial infinities by using complex horizontal slowness for each plane wave, mimicking the effect of geometrical spreading and surface wave attenuation (see the Appendix).

Figure 11 shows that the location of the horizontal component spectral peak depends on the input horizontal wave slowness. Slowly propagating $SH$ waves will resonate at lower frequency or larger periods that also happen to be the conditions for Love-wave dispersion in the layer. Slowly propagating $SV$ waves can show resonant peaks both at lower and higher frequencies, depending on the horizontal slowness. Note that this calculation is different from the simple constructive interference relations (equations 2 and 3) because the input waves are inhomogeneous waves and reflections at the base of the layer induce complex, postcritical angle phase changes. As the horizontal phase velocity gets close to the basal layer shear-wave velocity, the spectral peak stabilizes around the expected peak frequency determined from equation (3) and the amplifications gradually become smaller. As horizontal phase velocity increases, wave energy can refract out of the layer, considerably reducing the amplification effect. From this purely plane wave point of view, it appears that spectral resonances of horizontal component waves generally occur at lower frequency (longer period) than that predicted from equation (2) or (3).

A more realistic representation of the effect of surface wave interactions with a multilayered medium can be found from basic propagator matrix theory. Using upward continuation, a very simple relationship can be found for the $P - SV$-wave propagation system that gives the theoretical $H/V$ ratio at the surface, $\hat{r}_0(\omega)$, in terms of the $H/V$ ratio at depth, $\tilde{r}(\omega)$ (see the Appendix), in particular,

$$\hat{r}_0(\omega) = \frac{a_{22}\tilde{r}(\omega) - a_{12}}{a_{11} - a_{21}\tilde{r}(\omega)},$$

where the $a_{ij}$ are elements of the Thomson–Haskell product layer matrix. For the coordinate system shown in the Appendix (positive vertical downward and positive radial motion away), the $H/V$ ratio for a fundamental mode Rayleigh wave in a half-space can be defined as

$$\hat{r}(\omega) = -i\varepsilon,$$

where $\varepsilon$ is the ellipticity or magnitude of the $H/V$ ratio. The factor of $-i$ in the frequency domain gives retrograde elliptical motion in the vertical/radial plane for the Rayleigh wave in the time domain.

Figure 12 shows a series of computations for a simple layer-over-half-space model that displays the principal characteristics of upward continuation of a fundamental mode Rayleigh wave into a low-velocity layer. As shown in the Appendix, plane wave upward continuation can mimic a propagating surface wave with cylindrical geometrical spreading and/or anelastic attenuation through the assumption of a complex horizontal slowness. This neatly reduces the large surface $H/V$ ratios at the spectral peaks to values in line with those that are observed. Complex horizontal wave slowness reduces the spectral amplitude only at the peaks (Fig. 12a) but does not change the peak frequency. The amplitude and location of $H/V$ spectral peaks are surprisingly insensitive to the assumed Rayleigh-wave slowness.
Rayleigh ellipticity does change the location of peak frequency (Fig. 12c), but it is not a large effect over a large change in ellipticity. Figure 12d shows that the location of the H/V peak is a combination of both amplification in the horizontal and vertical components and a null in the vertical component spectrum as suggested by Bodin et al. (2001). This is very similar to the characteristics of the observed spectra shown in Figure 9. At the vertical spectral null, the surface ellipticity undergoes changes from retrograde to prograde as pointed out by Konno and Ohmachi (1998), among others. Overall, upward continuation of a fundamental mode Rayleigh wave through a low-velocity layer results in the creation of higher mode Rayleigh waves represented by spectral resonance peaks with changes in ellipticity with frequency.

Figure 12 displays upward continuation results for multilayered and single-layered sediment models corresponding to Marked Tree and Mooring array sites. The location of the fundamental H/V spectral peaks near 0.25–0.2 Hz is not affected by model complexity and have values that can be predicted almost exactly by equation (3) using average layer slowness and thickness values. This is the principal result of this article in that the spectral peak is not strictly due to constructive interference of horizontally polarized shear waves but also to destructive interference of the vertical component of interfering $SV$ waves (Appendix).
The level of amplification of local earthquake shear waves. Surface wave motion, therefore, is not a good indication of the H component over the V component as waves propagate in the embayment. The basic assumption of amplification of seismic waves must be significantly less for local earthquake seisms propagating into the embayment and locally convert phases that constructively interfere in the low-velocity sediment for horizontal and vertical components but also destructively interfere for the vertical component of motion. The amplification implied by the H peak is due to efficient trapping of shear waves because they postcrically reflect from the basal interface. However, wave amplification must be significantly less for local earthquake shear waves that are produced in the crust under the sediments. Earthquake shear waves will have higher horizontal phase velocities that will allow waves to quickly leak out of the sediment layer. The H/V peak produced from ambient surface wave motion, therefore, is not a good indication of the level of amplification of local earthquake shear waves.

Discussion

It is remarkable that Nakamura’s method works so well in the embayment. The basic assumption of amplification of the H component over the V component as waves propagate up through the low-velocity structure is roughly correct and the approximation of constructive interference of vertically propagating S-plane waves is good because the sediment velocity is so low compared to the incident wave horizontal phase velocity.

Figure 2 shows the estimates of the average shear-wave velocity under Marked Tree and Mooring array sites plotted with results taken from Bodin et al. (2001). These estimates are consistent with the previous measurements but fall at the lower bound of the H/V interpretations. In addition, the error bars span about half of the width of both the previous H/V and the Sp conversion interpretations. A conservative interpretation of our present results implies that there could be considerable velocity variation throughout the embayment or that there might be time-dependent changes in the nature of the ambient noise field. However, the H/V data yield a relation that shows that the average shear-wave velocity increases with sediment thickness in the embayment, suggesting that velocity increases because of sediment compaction. Our results give some confidence for performing a larger empirical study of average shear-wave velocity throughout the embayment much like those performed for Mexico City in mapping out sediment thickness and resonant periods for earthquake shaking hazards assessments (Lermo et al., 1988; Lermo and Chávez-García, 1993, 1994a,b).

However, Figure 2 still displays a major seismological inconsistency. If the H/V data are giving a reasonable estimate of average shear-wave velocity, why do the S – Sp travel times imply much lower average velocities? The answer is actually simple but surprising. Figure 14 shows microearthquake data from a broadband station, PARM, and a short-period station near the Mooring array, MORT. These are typical data recorded by the Cooperative New Madrid Seismic Network and show clear S and Sp phases. An Sp phase from a single interface should be simple and be similar to the P-wave or S-wave pulse. The Sp phases displayed in Figure 12 have relatively long durations. Furthermore, if an average S-wave velocity is taken from the H/V result of Figure 2 and thickness from Dart and Swolfs (1998) (Fig. 1), the expected Sp time occurs midway into the Sp waveform. At PARM, a distinct Sp phase is seen at the expected time but the Sp phase, as a whole, starts about 0.25 sec earlier. The same is true at MORT, but the short-period instrument response tends to produce a narrowband signal that blends Sp arrivals together.

Our conclusion is that Sp conversions within the Mississippi embayment may start with converted phases that occur deeper in the Paleozoic sedimentary section where early Paleozoic clastic rocks form a moderate low-velocity zone (Fig. 10). If the first Sp conversion is picked as the Sp from the unconsolidated sediment interface, then the larger S – Sp time will yield a lower average shear-wave velocity given an assumed layer thickness. Using the first-arrival Sp times on Figure 14 yields 481 m/sec for PARM, which has 400 m thick sediments, and 507 m/sec for MORT at 700 m thickness. These estimates are consistent with the Sp data shown in Figure 2 and strongly suggest that the
microseisms for two 1 hr time periods with our array experiment. Motions and the H/V signal in southern California displayed stable over time. Tanimoto (2006) showed that particle motions and the H/V signal in southern California displayed seasonal changes that suggested different source areas exciting different modes for the microseisms. Because the geological structure of the coastal plain sediments in the Mississippi embayment opens south and deepens to the Gulf of Mexico, microseisms produced by heavy storm surf on the Gulf coast could look significantly different than microseisms from the North Atlantic. Source–receiver distances would be significantly different allowing the observation of different wave types (e.g., Gerstoft et al., 2006). Velocity structure along southern azimuths would also be significantly different, probably producing low-phase velocity surface waves propagating in the thicker sediments. It is possible that H/V peaks are significantly different for microseisms traveling paths from the Gulf of Mexico compared to those from the North Atlantic.

Conclusions

The empirical assumptions that form the basis of Nakamura’s method are seen to approximately hold for producing the long-period H/V spectral peak in deep embayment sediments. Two array experiments show that ambient ground motions near the fundamental H/V peak are composed of relatively high-velocity Rayleigh and Love waves produced by high-storm surf along the northeastern coast of North America. H and V spectra measured simultaneously at a station outside the Mississippi embayment are identical and are consistent with Nakamura’s empirical assumption that these motions are equal before interacting with the sediments. The low-velocity embayment sediments cause inhomogeneous shear waves from the incident Rayleigh- and Love-wave trains to convert to homogeneous, near-vertically propagating shear waves within the sediments that then constructively interfere to produce a resonant peak in both horizontal and vertical motions but destructively interfere at a different frequency for vertical motions. The overall amplification is due to the fact that incident waves are inhomogeneous and that the converted homogeneous shear waves become completely trapped in the sediment layer. Amplification can be expected to be much less for higher phase velocity shear waves from local earthquakes.

Nakamura’s method appears to be a robust technique that can be used to perform an inexpensive study of the average velocity structure of the entire Mississippi embayment given previous knowledge of sediment thickness. The disagreement between average velocities determined by the H/V technique and differential S-p-phase times seen in previous studies is resolved by the observation that S-p phases start from phase conversions at interfaces below the unconsolidated sediment boundary.

Data and Resources

Software packages Generic Mapping Tool (GMT) (Wessel and Smith, 1998) and Seismic Analysis Code (SAC) (Goldstein et al., 2002) were used in this study and are gratefully acknowledged. Seismic data for station WVT (Waverly, Tennessee) were obtained from the Incorporated Research Institutes for Seismology (IRIS) Data Management Center. Mooring and Marked Tree array data will also be archived at the IRIS Data Management Center as part of the PASSCAL embayment seismic excitation experiment.
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References


Appendix

This article uses three theoretical results that need to be motivated with a rigorous discussion of plane wave propagation in a plane-layered system. These include upward continuation of the wave field, upward continuing the H/V ratio and its detailed behavior, and the use of complex-valued horizontal slowness in a 2D computation to simulate surface wave propagation from a point source. The formulation is closely related to the 2D theory outlined in Haskell (1962) and Harkrider (1971) but uses wave slowness rather than phase velocity and explicitly considers displacements from propagating P and S plane waves. As such, this is not new theory, but the final relationships could have some use in data analysis, particularly in continuing plane wave fields upward or downward in a stack of layers.

Upward Continuation

Consider a series of n plane, elastic layers over a half-space (the n + 1 layer) bounded at the top with a free surface. A Cartesian coordinate system with horizontal direction, x1 to the right, and vertical direction, x3 downward, has its origin at the free surface. Except for the free surface, all boundaries are welded so that horizontal and vertical displacements are continuous at each boundary as are normal and shear stress on each boundary. Fourier transformed Cartesian displacements, stresses,

\[ \hat{u}_1 = \frac{\partial \hat{\phi}}{\partial x_1} - \frac{\partial \hat{\psi}}{\partial x_3}, \quad \hat{u}_3 = \frac{\partial \hat{\phi}}{\partial x_3} + \frac{\partial \hat{\psi}}{\partial x_1}, \quad (A1) \]

and stresses,

\[ \hat{\tau}_{33} = \lambda \nabla^2 \hat{\phi} + 2 \mu \left( \frac{\partial^2 \hat{\phi}}{\partial x_3^2} - \frac{\partial^2 \hat{\psi}}{\partial x_1 \partial x_3} \right), \]

\[ \hat{\tau}_{31} = \mu \left( \frac{\partial^2 \hat{\phi}}{\partial x_1^2} - \frac{\partial^2 \hat{\psi}}{\partial x_1 \partial x_3} - \frac{\partial^2 \hat{\phi}}{\partial x_3^2} \right), \quad (A2) \]

are given in terms of scalar P and SV-wave potentials, \( \hat{\phi} \) and \( \hat{\psi} \), respectively. The homogeneous form of the plane wave potentials are given by

\[ \hat{\phi}(\omega) = \hat{A}_1(\omega) e^{-i\omega(p_3-x_3)} + \hat{A}_2(\omega) e^{-i\omega(p_3+x_3)}, \]

\[ \hat{\psi}(\omega) = \hat{B}_1(\omega) e^{-i\omega(p_3-x_3)} + \hat{B}_2(\omega) e^{-i\omega(p_3+x_3)}, \quad (A3) \]

where the amplitudes of upgoing waves are given by the “1” subscript and downgoing by the “2” subscript. All waves move to the right in the +x1 direction. Horizontal wave slowness is given by \( \eta \), and \( \omega \) is circular frequency. Vertical wave slowness is given by

\[ \eta_c = \left( \frac{1}{v^2} - \rho^2 \right)^{1/2}, \quad (A4) \]

where \( v = \alpha \) for P-wave velocity or \( v = \beta \) for S-wave velocity. The vertical slowness is multivalued when the horizontal slowness is greater than the medium slowness or is complex. In this case the appropriate branch for the vertical slowness is to take the negative sign for the complex square root.

The relationship between the coefficients of the plane waves and spectral displacements and stress are given by

\[ \begin{pmatrix} \hat{u}_1 \\ \hat{u}_3 \\ \hat{\tau}_{33} \\ \hat{\tau}_{31} \end{pmatrix} = \mathbf{D}(x_3) \begin{pmatrix} \hat{A}_1 \\ \hat{A}_2 \\ \hat{B}_1 \\ \hat{B}_2 \end{pmatrix}, \quad (A5) \]

where

\[ \mathbf{D}(x_3) = \begin{pmatrix} -i\omega \rho e^{-i\omega(p_3-x_3)} & -i\omega \rho e^{-i\omega(p_3+x_3)} \\ +i\omega \eta e^{-i\omega(p_3-x_3)} & -i\omega \eta e^{-i\omega(p_3+x_3)} \\ (i\omega)^2 \left[ \lambda \rho^2 + (\lambda + 2\mu) \eta^2 \right] e^{-i\omega(p_3-x_3)} & (i\omega)^2 \left[ \lambda \rho^2 + (\lambda + 2\mu) \eta^2 \right] e^{-i\omega(p_3+x_3)} \\ -i\omega \eta e^{-i\omega(p_3-x_3)} & +i\omega \eta e^{-i\omega(p_3+x_3)} \\ -i\omega \rho e^{-i\omega(p_3-x_3)} & -i\omega \rho e^{-i\omega(p_3+x_3)} \\ -(i\omega)^2 \mu \eta e^{-i\omega(p_3-x_3)} & +i\omega \mu \eta e^{-i\omega(p_3+x_3)} \\ -(i\omega)^2 \mu (\eta^2 - \rho^2) e^{-i\omega(p_3-x_3)} & -(i\omega)^2 \mu (\eta^2 - \rho^2) e^{-i\omega(p_3+x_3)} \end{pmatrix}, \quad (A6) \]
The horizontal propagation phase can be factored from equation (A6) through

\[
\begin{pmatrix}
\hat{u}_1 \\
\hat{u}_3 \\
\tau_{33}
\end{pmatrix} = N
\begin{pmatrix}
u \\
w \\
\sigma_R \\
\tau_R
\end{pmatrix}
\]

(A7)

where

\[
N =
\begin{pmatrix}
i\omega p e^{-i\omega p x_1} & 0 & 0 & 0 \\
0 & i\omega p e^{-i\omega p x_1} & 0 & 0 \\
0 & 0 & (i\omega)^2 e^{-i\omega p x_1} & 0 \\
0 & 0 & 0 & (i\omega)^2 e^{-i\omega p x_1}
\end{pmatrix}
\]

(A8)

The vertical functions of displacement and stress are then

\[
\begin{pmatrix}
u \\
w \\
\sigma_R \\
\tau_R
\end{pmatrix} = C(x_3)
\begin{pmatrix}
A_1 \\
A_2 \\
B_1 \\
B_2
\end{pmatrix}
\]

(A9)

where

\[
C(x_3) =
\begin{pmatrix}
-e^{-i\omega \eta_x x_3} & -r_\beta e^{i\omega \eta_x x_3} & +r_\beta e^{-i\omega \eta_x x_3} \\
r_\alpha e^{i\omega \eta_x x_3} & -r_\alpha e^{-i\omega \eta_x x_3} & -r_\beta e^{-i\omega \eta_x x_3} \\
+\gamma e^{i\omega \eta_x x_3} & +\gamma e^{-i\omega \eta_x x_3} & +\gamma e^{i\omega \eta_x x_3} \\
-\delta_\alpha e^{i\omega \eta_x x_3} & -\delta_\alpha e^{-i\omega \eta_x x_3} & -\delta_\beta e^{-i\omega \eta_x x_3}
\end{pmatrix}
\]

(A10)

and

\[
\gamma = \rho(1 - 2\beta^2 p^2), \quad \delta_\alpha = 2\rho \beta^2 p^2 r_\alpha, \\
\delta_\beta = 2\rho \beta^2 p^2 r_\beta, \quad r_\alpha = \frac{\eta_\alpha}{\rho}, \quad r_\beta = \frac{\eta_\beta}{\rho}.
\]

(A11)

These relations can be used to find the propagator matrix to obtain the vertical displacement/stress at \( z \) from that at \( z_0 \), where \( z > z_0 \):

\[
A_R = C(x_3 = z)C^{-1}(x_3 = z_0)
\]

(A12)

and

\[
\begin{pmatrix}
u \\
w \\
\sigma_R \\
\tau_R
\end{pmatrix}_{x_3 = z} = A_R
\begin{pmatrix}
u \\
w \\
\sigma_R \\
\tau_R
\end{pmatrix}_{x_3 = z_0}
\]

(A13)

Given displacement and stress at the top of the half-space below the system of \( n \) layers, the displacement and stress at the free surface is

\[
\begin{pmatrix}
u \\
w \\
\sigma_R \\
\tau_R
\end{pmatrix}_{x_3 = 0} = A
\begin{pmatrix}
u_{0} \\
w_{0} \\
\sigma_{R0} \\
\tau_{R0}
\end{pmatrix}
\]

where

\[
A = A_{R_n}A_{R_{n-1}} \cdots A_{R_2}A_{R_1}
\]

(A15)

is the product of all layer matrices of the layered stack.

The free surface spectral displacements are easily found from equation (A14) and are

\[
u_0 = \frac{a_{22} u - a_{12} w}{a_{22} a_{11} - a_{12} a_{21}}, \quad w_0 = \frac{a_{11} w - a_{21} u}{a_{22} a_{11} - a_{12} a_{21}}.
\]

(A16)

where (for a single layer of thickness \( d \))

\[
a_{11} = 2\beta^2 p^2 \cos \omega \eta_x d + (1 - 2\beta^2 p^2) \cos \omega \eta_x d, \\
a_{22} = (1 - 2\beta^2 p^2) \cos \omega \eta_x d + 2\beta^2 p^2 \cos \omega \eta_x d, \\
a_{12} = -\frac{(1 - 2\beta^2 p^2)}{r_\beta} i \sin \omega \eta_x d + 2\beta^2 p^2 r_\beta i \sin \omega \eta_x d, \\
a_{21} = -2\beta^2 p^2 r_\alpha i \sin \omega \eta_x d + \frac{(1 - 2\beta^2 p^2)}{r_\beta} i \sin \omega \eta_x d.
\]

(A17)

**Upward Continuation of the H/V Ratio**

Define the spectral H/V ratio at the half-space boundary as

\[
\tilde{r}(\omega) = \frac{u}{w}.
\]

(A18)

Taking the ratio of \( u_0 \) and \( w_0 \) using equations (A16) gives

\[
\tilde{r}_0(\omega) = \frac{u_0}{w_0} = \frac{a_{22} \tilde{r}(\omega) - a_{12}}{a_{11} a_{21} - a_{12} a_{21}}.
\]

(A19)

Now, for circularly polarized Rayleigh-wave motion at the top of the half-space \( \tilde{r}(\omega) = -i \). For a layer with low \( P \) - and \( S \)-wave velocities such as used in the computations in this article, it can be shown that the contribution from terms containing propagating \( P \) waves in the \( a_{ij} \) (terms with \( \cos \omega \eta_x d \) or \( \sin \omega \eta_x d \)) are small and that propagating \( S \) waves dominate the vertical and radial displacements. The zeros of the surface vertical displacement are given by

\[
a_{11} + i a_{21} = 0,
\]

(A20)

which, using equations (A17), is approximately

\[
(1 - 2\beta^2 p^2) \cos \omega \eta_x d - \frac{(1 - 2\beta^2 p^2)}{r_\beta} i \sin \omega \eta_x d = 0.
\]

(A21)

This gives

\[
r_\beta = \tan \omega \eta_x d.
\]

(A22)
But in terms of the real angle of incidence, $j$, for the propagating $S$ wave in the layer,

$$ r_\beta = \frac{\eta_\beta}{p} = \frac{\cos j}{\beta} \frac{\beta}{\sin j} = \cot j = \tan \left( \frac{\pi}{2} - j \right). \tag{A23} $$

So, from equation (A23), zeros occur at frequencies

$$ \omega = \frac{(\frac{\pi}{2} - j)}{\eta_\beta d} = \frac{(\frac{\pi}{2} - j) \beta}{\cos j d} \approx \frac{\pi \beta}{2d} \tag{A24} $$

for small $j$. Converting circular frequency to period gives the usual equation for constructive interference of shear waves in a layer except we have found the case for destructive interference in the vertical component:

$$ T = \frac{4d}{\beta}. \tag{A25} $$

Numerical experiments show that this result is robust for a large range of horizontal slowness greater than the shear-wave slowness in the half-space for upward continuation of Rayleigh-displacement fields.

Use of Complex Horizontal Slowness for Cartesian Plane Wave Simulations

Harkrider (1971) showed that the Fourier transformed displacement for Love or Rayleigh waves propagating in plane-layered structures has the form

$$ \hat{U}(\omega) \propto e^{-i\omega^{\frac{\phi}{r^2}}, \tag{A26} $$

where $r$ is the distance from the source and $c$ is the surface wave phase velocity. In addition, attenuation, or the surface wave quality factor, $Q$, can be approximately included in equation (A26) by an additional factor (Ben-Menahem and Singh, 1981)

$$ e^{-\frac{\pi}{Q} r}. \tag{A27} $$

A horizontally propagating plane wave has the form

$$ e^{-i\omega px}. \tag{A28} $$

Without loss of generality, at each frequency a complex horizontal slowness can be defined to match the amplitude decay and horizontal phase velocity of a surface wave. This is done by equating the plane wave and surface wave displacements:

$$ e^{-i\omega px} = e^{-i\omega \frac{\phi}{r^2}} = e^{-i\omega \frac{\phi}{r^2} - \frac{\pi}{Q} r \ln r}. \tag{A29} $$

Allowing $p$ to be complex and letting $x_1 = r$ gives

$$ \text{Re} p = \frac{1}{c}, \quad \text{Im} p = -\frac{1}{2cQ} - \frac{1}{2} \frac{\ln r}{\omega r}. \tag{A30} $$

The geometrical spreading part of $\text{Im} p$ will probably dominate unless $Q$ for 4 sec surface waves are of the order 100 or less. The magnitude of the imaginary part will be in the range of $-0.001$ to $-0.005$ for 4 sec surface waves produced 1000–2000 km away from the embayment. We found that $\text{Im} p = -0.01$ was needed to produce amplitude ratios of the order 10 in the upward continuation computations, and because we have no information on how $Q$ might change with frequency, we assume a constant value for the imaginary part over the frequency band of interest. This could imply greater than expected geometrical spreading due to defocusing of the surface waves or low $Q$, or both. We do not want to overinterpret the use of complex horizontal slowness, but simply to point out that it is a method that can be used to examine complex wave propagation mechanisms with relatively simple plane wave theory.

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